Numerical simulation of the fatigue crack growth with history effects. Complex non-isothermal loading conditions.

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Abstract When complex loading conditions are encountered, the fatigue crack growth rate remains difficult to model because of various coupling effects. The problem that was considered in this research, is the growth of a crack at high temperature (450°C - 650°C) in a nickel base superalloy under variable temperature, stress amplitude and stress rates loading conditions. This material and other nickel base superalloys were widely studied in the past under isothermal conditions [4,5,8,9,10]. The mechanisms leading to dwell time effects at various temperatures, for instance, are well understood. The main objective of this research is to use this knowledge to model the fatigue crack growth rate under complex "missions". An incremental model was previously developed specifically to take into account variable amplitude loading conditions. In this research, this model was extended so as to consider also variable stress rates and non-isothermal loading conditions.

1 INTRODUCTION

The aircraft industry face up the problem of predicting fatigue fracture of their aircraft engines parts under variable loads, variable thermal conditions and detrimental environmental conditions (moist and propitious to oxidation). A more accurate prediction of the fatigue crack growth rate under complex loading conditions is useful to improve the inspections intervals of industrial components. Predicting fatigue crack growth in metals under random loadings remains difficult since the fatigue crack growth rate is very sensitive to load history.

Constrained plasticity at crack tip is known for decades to induce history effects in fatigue crack growth. Moreover, these effects are closely related to the cyclic elastic-plastic behaviour of the material. Therefore, a strategy was proposed [6,7] so as to identify a history-dependent fatigue crack growth model at the global scale, using detailed finite element computations that include suitable material constitutive equations. This model is a set of about ten scalar derivative equations and allows therefore computing a million of variable amplitude fatigue cycles within a few seconds. In this model, the crack growth rate is a time derivative equation *da/dt*, therefore its explicitly avoids the need of a cycle counting method. Besides, it facilitates largely the writing of coupled multi-physic problems, such as oxidation assisted fatigue crack growth and non-isothermal fatigue. This model was identified and validated for a low carbon steel at room temperature using constant and variable amplitude loading conditions [6,7]. The aim of the present research [2,3] is to examine how it can be extended so as to consider oxidation assisted fatigue crack growth and non-isothermal loading conditions.

2 MULTISCALE STRATEGY

Let us briefly summarize what has already been done for mode I fatigue crack growth under variable amplitude loadings [6,7]. The fatigue crack growth model is based on the assumption that the fatigue crack growth rate is function of crack tip plasticity. Therefore the model is divided into two parts. The first part provides the fatigue crack growth rate as a function of the rate of variation of a global measure of plastic deformation in the crack tip region. The second part allows predicting the rate of variation of that global measure of plastic deformation in the crack tip region according to the loading conditions and of the values of a set of internal variables introduced so as to account for history effects.

The second part of the model can be considered as a global cyclic elastic-plastic constitutive model for the crack tip region. It was developed within the framework of dissipative processes. So as to benefit on the one hand from the ability of local FE model to account for detailed and complex material behaviours and on the other hand of the computational efficiency of global crack growth criteria, a multiscale strategy was proposed. It consists merely in assuming that the displacement field in the crack tip region is partitioned into elastic and plastic parts.

$$\underline{u}(\underline{x},t) \approx K_I^*(t)\,\underline{u}_I^e(\underline{x}) + \rho_I(t)\underline{u}_I^c(\underline{x}) \tag{1}$$

Each part is also assumed to be the product of an intensity factor and of a reference field function only of spatial coordinates. When the material behaviour is elastic, the intensity factor of the elastic field K_I^* is, as usual, the stress intensity factor K_I. When crack tip plasticity occurs, it diverges slightly from K_I but the difference remains very small and will be neglected. The intensity factor of the plastic part, noted ρ, will be called the plastic strain intensity factor. Numerically, it is always possible to perform an approximation such as that in Eq. 1, using, for instance, the Karhunen-Loeve transform. In practice, u_I^e is the finite element solution of an elastic problem with boundary conditions such as such as that $K_I = 1$ MPa.m$^{1/2}$. Then an elastic-plastic finite element computation is performed, with K_I increasing from zero to a maximum value that is consistent with a fatigue application (i.e. a loading ramp from 0 up to 40 MPa.m$^{1/2}$ for instance). At the end of the computation, K_I^* is obtained by projecting the elastic-plastic numerical solution onto the reference elastic field u_I^e. The rest is divided into an intensity factor and a reference field using the Karhunen-Loeve transform. This allows building a numerical solution for u_I^c. At this stage, the reference fields u_I^e and u_I^c are known and are used to extract ρ(t) from any future elastic-plastic FE computations. Besides, an analytical approximation for u_I^c can be proposed [6] using the theory of distributed dislocations, it can be considered as the displacement field around an edge dislocation aligned with the crack plane and with a Bürgers vector equal to 1.

Once the reference fields u_I^e and u_I^c are known, the approximation in Eq. 1 is performed at each step of a FE computation using a post-treatment routine based on the least square method. The mean square error between the computed displacement field u(x,t) and the approximated one $K_I^*(t)u_I^e(x)+\rho_I(t)u_I^c(x)$, is always computed and remains below 10% under typical fatigue loading conditions.

Thus, the plastic strain intensity factor ρ is a global measure of plastic deformation in the crack tip region. The finite element method and the post-treatment routine are employed so as to generate evolutions of ρ under various loading conditions, including cyclic loadings with a stationary (Fig. 1) or a growing crack. Then an empirical model, developed within the framework of dissipative processes is associated to these evolutions and allows predicting $d\rho/dt$ as a function of dK_I/dt and of a set of internal variables [6,7]. For instance, at each load's reversal (Fig. 1), it is observed that there is a domain within which no variation of ρ is observed. Since ρ is a measure of crack tip plasticity, this domain (E in Fig. 1) can be considered as an elastic domain for the crack tip region. This domain moves (Fig. 1). Therefore a first internal variable can be introduced so as to define its position. Since its size varies, a second internal variable is introduced so as to define its size.

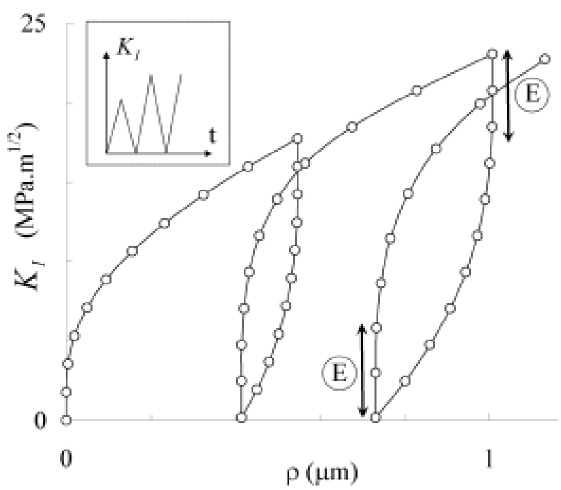

Figure 1. The nominal stress intensity factor K_I is plotted against the plastic strain intensity factor ρ as calculated using FE results of the simulation of a cyclically increasing load sequence. Within the domain (E) the variation of ρ is negligible; the cracked structure is assumed to behave elastically.

Then, empirical evolution laws are also introduced for these internal variables functions of the rate of variation of ρ and of the crack growth rate. These evolution laws are identified using FE analyses. The material parameters in these equations are now found using an automated procedure. Finally, the model consists in about ten scalar partial derivative equations, and provides dρ/dt as a function of the loading rate, of the current value of KI and of the internal variables V_{int} that stand for defining the plasticity induced history and of the material parameters P^i that can be identified for each material using the multiscale strategy proposed above:

$$\frac{d\rho_I}{dt} = f\left(\frac{dK_I}{dt}, K_I, V_{\text{int}}^j, P^i\right) \tag{2}$$

Finally, a crack propagation law was introduced which states that the crack growth rate is function of the rate of variation of the plastic strain intensity factor:

$$\frac{da}{dt} = \alpha\left|\frac{d\rho}{dt}\right| \tag{3}$$

Gathering the global cyclic elastic-plastic constitutive model for the crack tip region, that predicts dρ/dt as a function of the loading conditions and history and the crack propagation law that predicts da/dt as a function of dρ/dt yields finally a crack propagation model which accounts for plasticity induced history effects. This approach is operational for mode I fatigue crack growth, and, out of its numerical efficiency, it has other advantages such as avoiding cycle counting. The model was identified for a low carbon steel by R. Hamam et al. [7] using push-pull tests and constant amplitude fatigue crack growth experiments at R=0, and validated using other stress ratios and variable amplitude fatigue crack growth experiments.

3 NON ISOTHERMAL LOADING CONDITIONS

One advantage of a time-derivative formulation of the fatigue crack propagation law is that it facilitates the writing of the crack extent during missions for which the temperature varies. As a matter of fact, by now, if the temperature varies during the fatigue cycle, one reference temperature should be defined for the whole cycle. Usually, so as to get conservative predictions, the reference temperature is the maximum temperature encountered during the mission. However, this approach is over-conservative if, for instance, the maximum of the temperature is reached at minimum stress. If, the crack growth rate is defined as a time-derivative equation this problem vanishes.

However, the model should account for temperature variations. The problem is divided into two sub-problems. First of all, the history effects induced by plasticity and measured at the global scale through the internal variables V_{int}, should be functions of the temperature. This part of the problem is related to the evolution of the cyclic elastic-plastic behaviour of the material with the temperature. Secondly, the crack propagation law is also expected to be function of the temperature and of its history.

3.1 Global cyclic elastic plastic model for the crack tip region for non-isothermal conditions.

First of all, the constitutive behaviour of the N18 nickel base superalloy was identified at 450°C, 550°C, 600°C and 650°C on the basis of a database of experimental results produced at Snecma and by other researchers [9,10]. A non-linear kinematic hardening model was employed for each temperature. The hardening rate and the size of the elastic domain drop down when the temperature is increased. But, as it can be seen in Fig. 2, the variations of the material constitutive behaviour remain moderate between 450°C and 650°C. Besides, since it was observed [9] also that there is no dwell time effect on fatigue crack growth under vacuum conditions at 650°C, while a large effect is observed in air, creep crack growth is considered to be less important than oxidation assisted fatigue crack growth. Therefore, the material behaviour was not modelled as elastic-vicoplastic, but as elastic-plastic, even at 650°C.

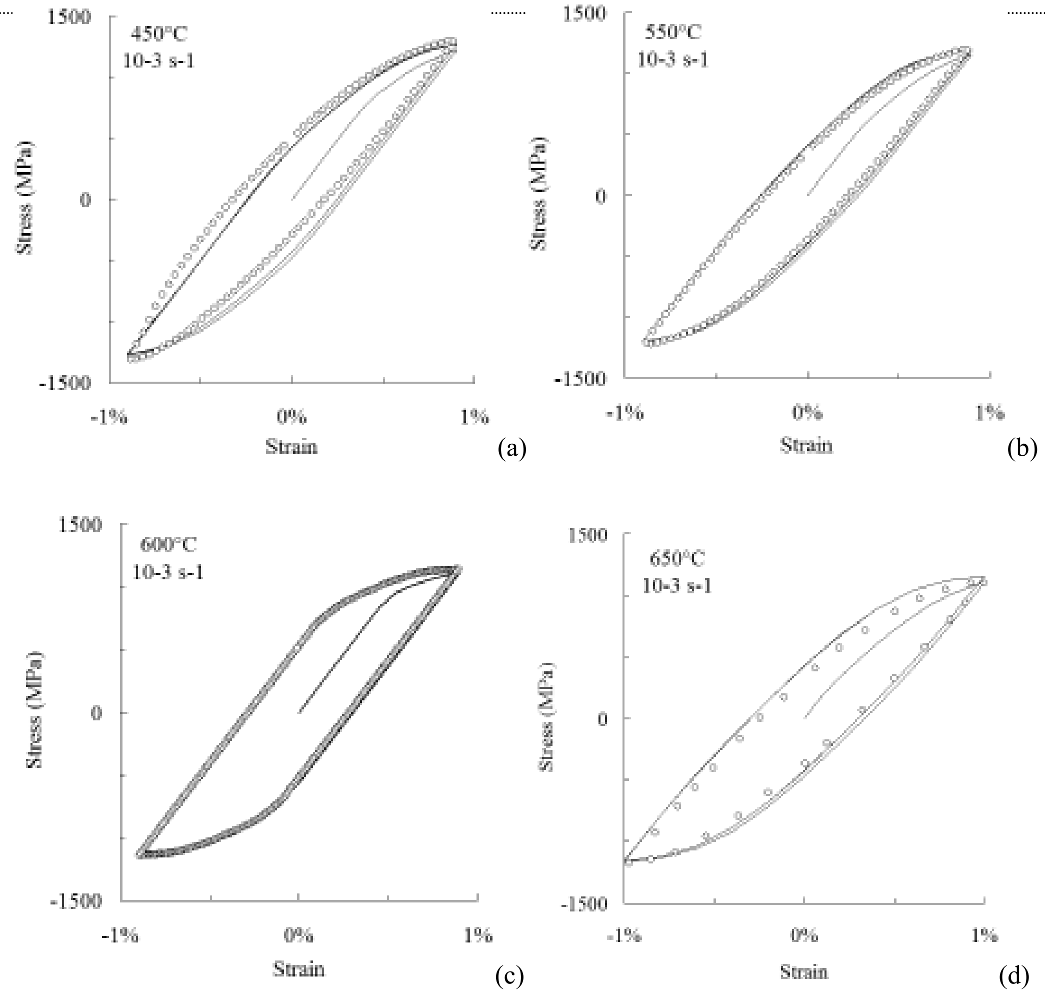

Figure 2: Comparison between experimental and simulated push-pull tests on the N18 nickel base superalloy at (a) 450°C, (b) 500°C, (c) 600°C and (d) 650°C

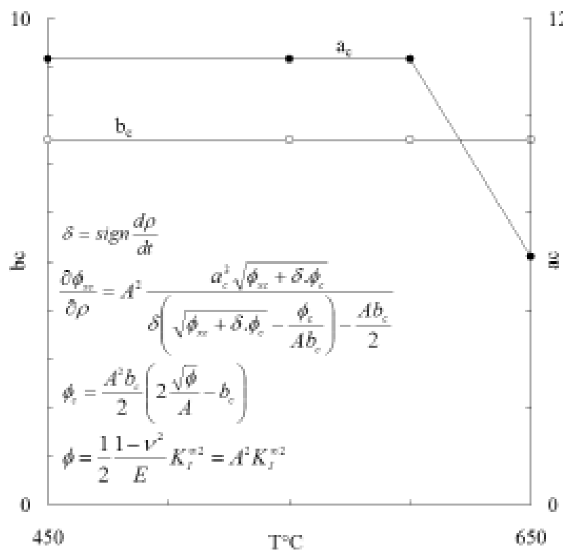

Figure 3: Example of material parameter evolutions. The parameter b_c that defines the size (denoted as ϕ_c) of the elastic domain of the cyclic plastic zone (E in Fig. 1) is constant. The parameter a_c that defines how this elastic domain is displaced decreases above 600°C. n.b. The current position of the elastic domain is stored in the internal variable ϕ_{xc}. The loading condition is defined by ϕ (ϕ=G_I/2).

Then for each temperature, using the multiscale approach discussed above, the material parameters (P_i) of the global cyclic elastic-plastic constitutive model *f* are identified. The equations that are employed in this model were chosen so as to ensure the identification of a unique set of material parameters (P_i) for each condition, which ensures that the interpolation between the temperatures is possible. As a consequence, by now, the global cyclic elastic-plastic constitutive model for the crack tip region contains a set of material parameters that are defined as piecewise linear functions of the temperature (Fig. 3).

$$\frac{d\rho_I}{dt} = f\left(\frac{dK_I}{dt}, K_I, V_{\text{int}}^j, P^i(T)\right) \tag{4}$$

3.2 Crack propagation law.

Besides, a crack propagation law is also introduced that defines the rate of creation of cracked area per unit length of the crack front (*da/dt*) as a function of dρ/dt. It is typically proposed that the crack propagation rate obeys Eq. 3, where α is a parameter to be adjusted using a high frequency and constant amplitude fatigue crack growth experiment (sinus, R=0.05, *f*=0.5 Hz).

This approach was applied to mode I fatigue crack growth at a moderate temperature (550°C) and validated using constant amplitude and complex fatigue crack growth experiments. At 550°C the effect of dwell time (Fig. 4) is negligible. As a matter of fact, the fatigue crack growth rate using triangle cycles at 0.5 Hz or cycles including a 300 seconds dwell time (10-300-10) is nearly the same. By comparison, the fatigue crack growth rate obtained using complex cycles is much higher. These complex cycles are constructed as follows, after a 10 seconds loading ramp, the sample is partially unloaded (by x% = 0%, 10%, 20%) and then 5 or 10 secondary cycles are applied, before the sample is unloaded. This type of cycles was also employed, for instance, by Chassaigne et al. [10]. Various combinations have been tested and the model always yields satisfactory results.

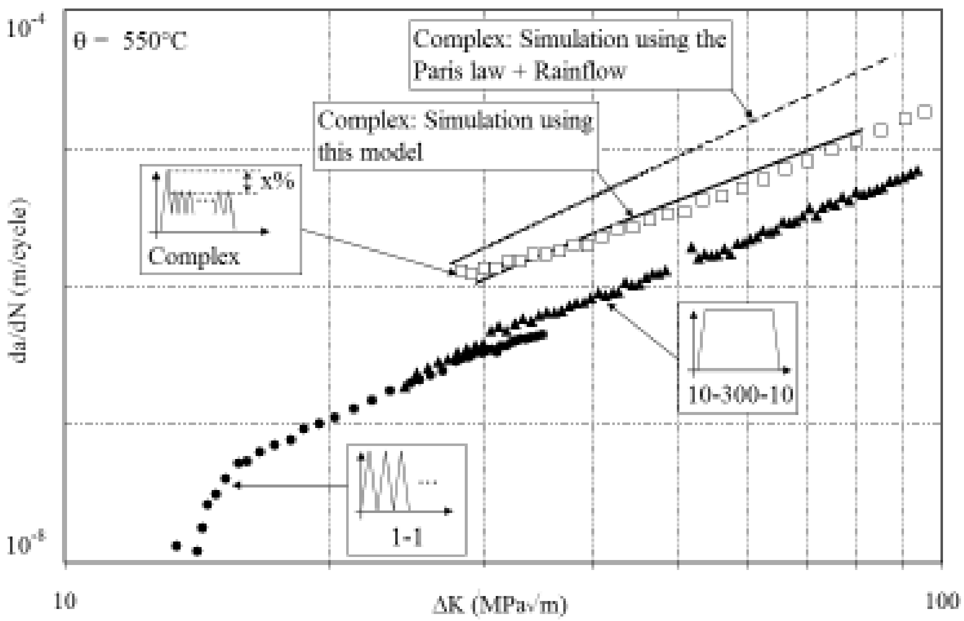

Figure 4: Fatigue crack growth rate at 550°C in the N18 nickel base superalloy. Experiments correspond to symbols: 1-1 cycles, 10-300-10 cycles and complex cycles with 10 sec loading, 20% of unloading, 10 secondary cycles and 10 sec unloading. The dotted line corresponds to the prevision for the complex cycle using the Paris law and Rainflow counting. The thick line corresponds to the prevision using the present model.

It is interesting to compare the predictions of this model with what would be predicted using the Paris law and a Rainflow counting method, so as to account for the contribution of secondary cycles. The predicted crack growth rate by such a classical approach is far over the experimental one. As a matter of fact, a secondary cycle is considered as fully efficient, since its stress ratio is high enough to keep the crack open. However, using the present model a large part of a secondary cycle is considered as inefficient. As a matter of fact, the crack growth rate is proportional to the plasticity rate, therefore when dp/dt is close to zero the cycle is inefficient. This happens when the cycling amplitude is close to the size of the elastic domain (E in Fig. 1). With the present model the cycle is inefficient when the crack is closed and also when the region at crack tip remains essentially elastic. Introducing an elastic domain is therefore analogous to introducing an intrinsic non propagation threshold.

However, since fatigue crack growth at high temperature is aimed to be modelled, the phenomenon of oxidation that assists fatigue crack growth should also be considered. This mechanism is responsible, for instance, of the detrimental effect of dwell times at higher temperature. As a matter of fact, the grain boundaries are oxidized ahead of the crack tip and it is observed that the crack path changes from transgranular to intergranular when a dwell time at maximum stress is added to the fatigue cycle. This mechanism is thermally activated. Besides, the material is designed to develop a passivation layer of oxides which protects the material against grain boundary oxidation. This second mechanism is also thermally activated.

Thus, at the beginning of a dwell time, grain boundary oxidation takes place at crack tip and the crack propagates, but as soon as the passivation layer has grown this propagation stops. It was shown, by Chassaigne et al. that only the very first seconds of a 300 sec dwell time contribute to crack growth. However, if the crack tip is stretched the passivation layer breaks, and the competition between grain boundary embrittlement and the growth of a passivation layer takes place again. Therefore this competition is driven by the mechanical loading. This explains, in particular, why the crack growth rate is not only sensitive to the duration of the fatigue cycle but also to its shape. This was demonstrated for instance by Hochstetter et al. [9]. At 575°C, two types of cycle were used, slow-fast and fast-slow, with the same duration but a different shape. During a fast-slow cycle, the passivation layer is broken at loading, but the loading phase is short and the detrimental effect of the environment is small, at unloading the passivation layer grows within a few second and protects the material during the unloading phase. On the contrary, during a slow-fast cycle, the passivation layer breaks and grows continuously during loading and the loading phase is long enough so that oxidation can assist fatigue crack growth. It is observed therefore that the crack growth rate is around twice higher using slow-fast cycles than using fast-slow cycles (Fig. 5).

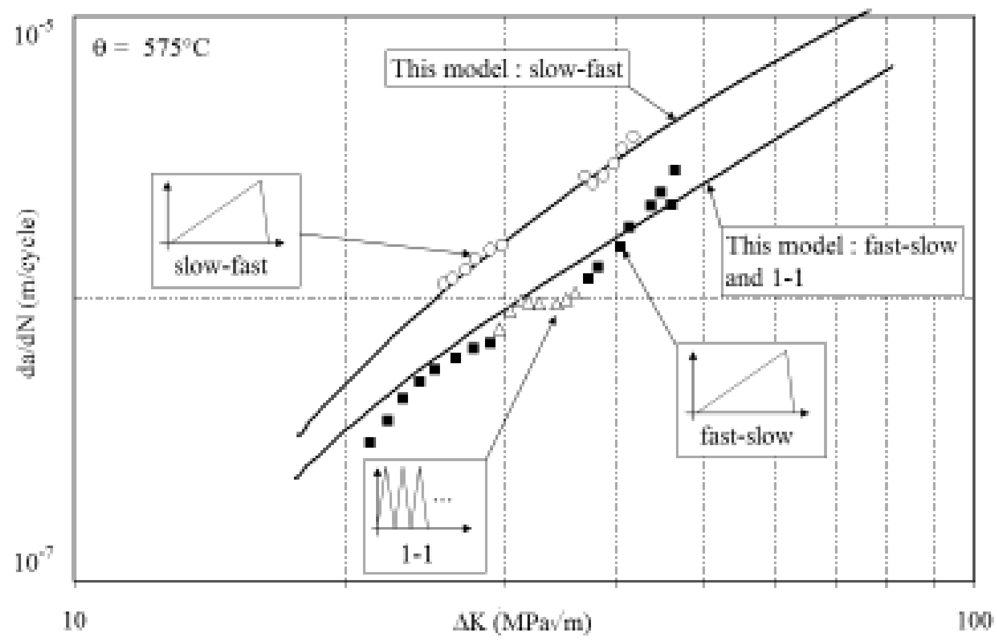

Figure 5: Fatigue crack growth rate at 575°C in the N18 nickel base superalloy [9].
Experiments correspond to symbols: a triangle 1-1 cycle, slow-fast and fast-slow cycles.
The thick lines correspond to the previsions using the present model.

These phenomena are modelled as follows. The crack growth rate is the sum of the contribution of crack tip plasticity and of the time during which grain boundary oxidation takes place :

$$\frac{da}{dt} = \left(\frac{\partial a}{\partial \rho}\right)\frac{d\rho}{dt} + \frac{\partial a}{\partial t} = \left(\alpha(T)\frac{d\rho}{dt}\right) + \frac{\partial a}{\partial t} \tag{5}$$

The global cyclic elastic-plastic constitutive model *f* that provides the plasticity rate as a function of the loading conditions and of the status of its internal variables, is a function of the temperature through the dependency of its material parameters (P_i) to the temperature. Besides, the adjustable parameter α was determined using high frequency fatigue crack growth experiments (sinus) for which the crack path is transgranular. These identifications were performed independently at 450°C, 550°C, 600°C and 650°C and interpolated.

Then, so as to account for stress rate variations (which include also the effect of dwell times), grain boundary embrittlement is modelled as follows:

$$\frac{\partial a}{\partial t} = m.\beta = m.\beta_o \exp\left(-\frac{Q}{kT}\right) \tag{6}$$

The crack growth rate β by grain boundary embrittlement, in absence of any passivation layer, is thermally activated and modulated by a variable *m* that stands for the state of the passivation layer. When m =1, the passivation layer is broken, when m =0, the passivation layer is thick enough to fully protect grain boundaries.

$$\frac{\partial m}{\partial t} = -R_G.m \qquad and \qquad \frac{\partial m}{\partial \rho} = -R_F.(1-m) \tag{7}$$

In between, two parameters R_G and R_F control respectively the rates of growth of that passivation layer versus time and its rate of fracturation when the crack tip is stretched due to crack tip plasticity. This happens only at opening, when dρ is positive. It is worth to underline that since the amplitude of ρ during a fatigue cycle increases when ΔK increases, the value chosen for R_F interferes with the slope of the simulated fatigue crack growth rate curve in the Paris diagram, which was useful to identify this parameter. Finally, using experiments including dwell times, various frequencies and complex cycles, it was possible to adjust β, R_G and R_F at each temperature. By interpolating the values of β obtained at 550°C, 600°C and 650°C by a thermally activated function, an activation energy Q was found empirically whose value is nearly that given in the literature for the activation energy of self-diffusion in nickel at grain boundary [8].

This model was validated using complex isothermal fatigue crack growth experiments. For instance, it was possible to reproduce the difference between fast-slow and slow-fast tests (Fig. 5) that were performed at a temperature intermediate between two identification temperatures. This results checks both the interpolation versus the temperature and the oxidation assisted crack growth part of the model. The model was also employed to simulate complex non-isothermal loading sequences, yielding to a good prediction of the total crack propagation life in an industrial test component.

4 CONCLUSIONS

An incremental model has been developed in order to predict fatigue crack growth under variable amplitude loading conditions. The aim of the present research was to examine how to extend the model to non-isothermal loading conditions and how to account for environmentally assisted cracking.

The rate of creation of cracked area per unit length of the crack front *da/dt* is assumed to be the sum of a term proportional to the rate of plastic deformation in the crack tip region and of a term accounting for environmentally assisted cracking.

The first part of the model is developed partly using the Fe method. As a matter of fact, a strategy is proposed to transfer to the global scale the very detailed data generated using local FE computations in the crack tip region. Then a simplified model is identified at the global scale and allows accounting for plasticity induced history effects. This part of the model requires, at various temperatures within the useful temperature range, the identification of the constitutive behaviour of the material (push-pull tests) and constant amplitude fatigue crack growth experiments. These tests should be conducted at a high enough frequency so as to avoid environmentally assisted cracking. Then the model was interpolated between the identification temperatures, so as to be able to model non-isothermal loading conditions.

The second part of the model aims at modelling the competition between the growth and fracture of a passivation layer and the grain boundary embrittlement which is believed to stem from oxygen diffusion along grain boundaries. This competition was modelled by a thermally activated cracking rate in absence of a passivation layer which is modulated by a parameter varying between 0 and 1, than stands for the state of the passivation layer. It grows up to one when the crack tip is stretched, because the passivation layer is assumed to be broken and goes down to zero progressively when time elapses, so as to account for the growth of the passivation layer. That model contains only three additional parameters for each temperature.

The model was applied to constant amplitude and complex isothermal fatigue tests and also to complex non-isothermal missions on industrial tests components giving satisfactory results.

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